

SKA SYSTEM BUDGET – LOW SYSTEM TIMING BUDGET

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TABLE OF CONTENTS

1 INTRODUCTION 7
1.1 Purpose of the Document 7
2 REFERENCES 8
2.1 Applicable Documents 8
2.2 Reference Documents 8
3 SUMMARY DESCRIPTION 9
3.1 System Synchronisation and Timing 9
3.2 Source L-0 Science Requirements: 10
3.3 Applicable Level-1 Requirements: 10
4 COHERENCE LOSS BUDGET 12
4.1 Summary Description 12
4.1.1 Coherence Loss 12
4.1.2 Budget Definition 12
4.2 Source L0 Requirements 13
4.3 Applicable L1 Requirements 13
4.4 Analysis Method 14
4.4.1 Relation to sensitivity on local interferometry 14
4.4.2 Phase fluctuations in Clock Distribution and Signal path 15
4.5 Calibration Steps and Effects 16
4.6 Summary of Margin and Residuals 16
4.6.1 Existing Risks 17
4.7 Source Errors 17
4.8 Budget Maturity Table 17
5 PHASE DRIFT BUDGET 18
6 SYNCHRONISATION AND TIME BUDGET 19
6.1 Summary Description 19
6.1.1 System Synchronisation and Time Budget 19
6.1.2 Budget Definition 19
6.2 Source L-0 Requirements 19
6.3 Applicable L1 Requirements 19
6.4 Analysis Method 20
6.4.1 Pulsar Timing 20
6.4.1.1 Precision of the offset time: σ_{TOA} 21
6.4.1.2 Precision of the absolute time reference: σ_0 21
6.4.2 Measurement of t_0 From the Timestamps of the Samples Coming From the Receivers and the Delay Model 21
6.4.3 t_0 Uncertainty Contributors 25
6.5 Calibration Steps and Effects 26
6.6 Summary of Margin and Residuals 26
6.6.1 Existing Risks 27
6.7 Source Errors 27
6.8 Budget Maturity Table 27

7 JITTER (REV.2) 28
7.1 Summary Description 28
7.2 Source L0 Requirements 28
7.3 Applicable L1 Requirements 28
7.4 Analysis Method 28
7.4.1 Definitions 28
7.4.1.1 **Signal to Noise Ratio (SNR):** 28
7.4.1.2 **N-bit Converter SNR**: 28
7.4.1.3 **Signal to Noise and Distortions (SINAD):** 28
7.4.1.4 **Effective Number of Bits (ENOB):** 29
7.4.1.5 **SNR Due to Jitter**: 29
7.4.1.6 **SNR Due to Jitter for a Flat Spectrum** 29
7.4.2 Discussion on the SNR for SKA1-Low 29
7.4.3 Discussion on Initial Jitter Specification 30
7.4.4 Updated Assumptions 30
7.4.5 Jitter 31
7.5 Calibration steps and effects 31
7.6 Summary of Margin and Residuals 32
7.6.1 Existing Risks 32
7.7 Source Errors 33
7.8 Budget Maturity Table 33
8 OUTSTANDING ISSUES, RISKS, TBDS AND TBCS 34

LIST OF FIGURES

Figure 1 System Timing Context 9
Figure 2 Local Interferometry for 2 Receivers with SADT.STFR Clock Distribution 14
Figure 3 Phase Fluctuation Components in a Single Station 15
Figure 4 Pulsar Timing Process 20
Figure 5 Telescope Architecture for Pulsar Timing with t0 Measurement From the Receivers Timestamps 22
Figure 6 Delay and Uncertainty Variables For the Analysis (To be Completed) 23

LIST OF TABLES

Table 1: Source L0 Science Requirements 10
Table 2 Applicable L1 Requirements 11
Table 3 L0 Requirements Driving Coherence Loss Budget 13
Table 4 Applicable L1 Requirements to Coherence Loss Budget 13
Table 5 SKA1-Low System Coherence Loss and Frequency Stability 16
Table 6 SKA1-Low Coherence Loss Budget Allocation 16
Table 7 Main Contributors For SKA1-Low Coherence Budget 17
Table 8 Secondary Contributors In SKA1-Low Coherence Budget 17
Table 9 SKA1-Low Coherence Loss Budget Maturity Table. 17
Table 10 L0 Requirements Driving Synchronisation and Time Budget 19
Table 11 Applicable L1 Requirements 20
Table 12 t_0 uncertainty contributors 25
Table 13 SKA1-Low Synchronisation and Time Budget Allocation 26
Table 14 Timing Dissemination and Delay Model Uncertainty Allocations 26
Table 15 SKA1-Low Synchronisation and Time Budget Contributors 27
Table 16 SKA1-Low Synchronisation and Time Budget Maturity Table 27
Table 17: Applicable L1 Requirements 28

1 Introduction

1.1 Purpose of the Document

The purpose of this document is to define and to specify the System Synchronisation and Timing Budgets for the Low telescope. General definitions for System budgets are provided by [AD1]. Initial identification for budgets is performed in [RD1].

The Low System Timing budget is composed by:

- Coherence Loss Budget
- Phase Drift Budget
- Timing Budget
- Jitter Budget

The document is structured as independent sections covering each of the previously outlined budgets. By the time of writing of this document version the Phase Drift Budget is not developed.

Each budget in the document is organized by first presenting a high level definition and driver requirements, following an analysis is applied for the budget definitions. Finally, budget contributors, calibration, margin, risks, source errors and maturity are captured.

2 References

2.1 Applicable Documents

The following documents are applicable to the extent stated herein. In the event of conflict between the contents of the applicable documents and this document, **the applicable documents** shall take precedence.

[AD1] SKA System Engineering Management Plan SKA-TEL-SKO-0000024 Rev. 02
[AD2] SKA1 Phase 1 System Requirement Specification SKA-TEL-SKO-0000008 Rev. 11

2.2 Reference Documents

The following documents are referenced in this document. In the event of conflict between the contents of the referenced documents and this document, **this document** shall take precedence.

[RD1] SKA-TEL-SKO-0000294_01 SKA1 System Budget Definitions.
[RD2] 100-000000-001 Rev.3 SKA_Low Functional Architecture
[RD3] Coherence Loss and Delay Observation Error in Very-Long Baseline Interferometry (Noriyuki, 1983) rd2
[RD4] SKA-TEL-SADT-0000499_04 SKA1 Level 1 Synchronisation and Timing Requirement Analysis and Verification (SADT unreleased draft) rd3
[RD5] Handbook of Pulsar Astronomy (Lorimer & Kramer, Cambridge University Press, 2005) r10
[RD6] Essential Radio Astronomy (https://science.nrao.edu/opportunities/courses/era/) Rd11
[RD7] SKA-TEL-SADT-0000011 SADT.SAT.CLOCKS-DD-Rev04-SKA1_TimeScale_Design_Report rd4

[RD8] "Analog-Digital Conversion Handbook (2004)" (Analog Devices)

[RD9] Interface Control Document SADT-LFAA (100-000000-026) Rev. 03.

3 Summary Description

3.1 System Synchronisation and Timing

The System Synchronisation and Timing budget will describe contributors to synchronisation and timing errors of several sources, including UTC traceability, time stamping, clock accuracy and clock stability. The latter of these directly contribute to system noise and telescope sensitivity, however longer-term stability behaviour (and traceability) are critical for Pulsar timing science.

According to [RD2] the SAT system and LFAA provide 3 system functions:

- Provide Reference Frequency/Sampling Clock:
  - Contributions under L1 requirements/budget: Phase stability (coherence and phase drift).
  - The Sampling clock is also characterised by Jitter, for Jitter there are no L1 requirements related to timing but spectral and digitization requirements for LFAA can specify jitter.
- Provide Synchronisation pulse: 1PPS synchronous pulse (locked to UTC)
  - Contributions under L1 requirements/budget: Precision (uncertainty).
- Provide Absolute Time: UTC time
  - Contributions under L1 requirements/budget: Phase relative to UTC (offset) and precision (time uncertainty).
  - Not under L1 requirements: Use of network time protocols and associated accuracy. (kind of L2 implementation).

Figure 1 below provides a context diagram for the Timing System and its functions in SKA1-Low.

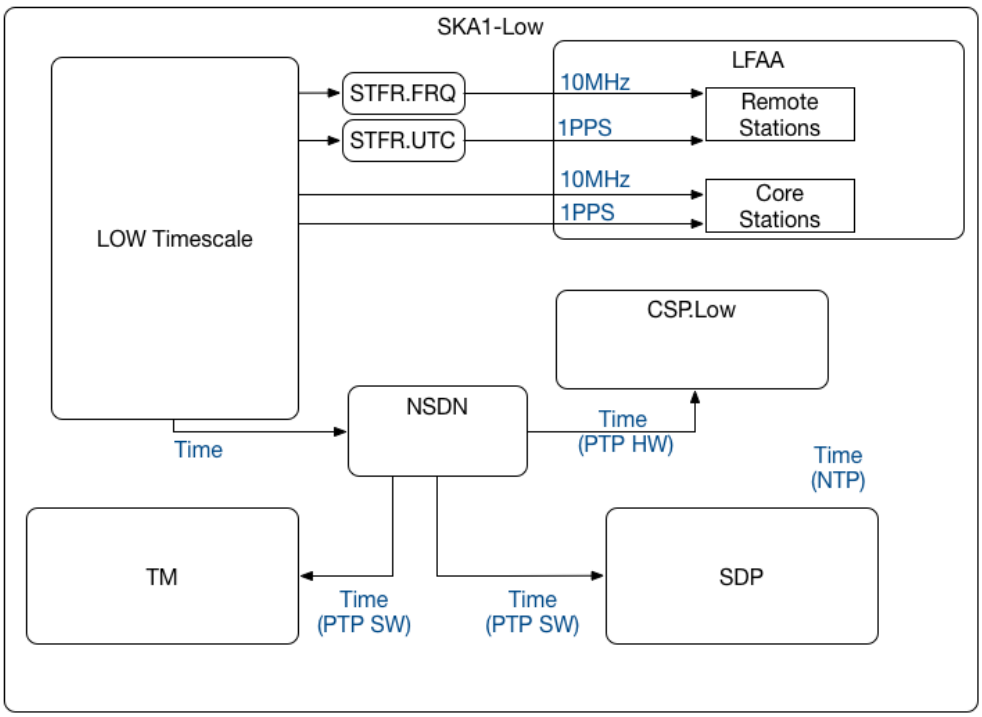


Figure 1 System Timing Context

The identified system budget components are:

- Science Driven:
    - Coherence Loss
    - Phase Drift
    - Synch & Time
- Engineering Driven:
    - Jitter (No L1 requirement for this)

As described in Figure 1 these budgets are shared between SADT (SAT) and LFAA for Coherence Loss, Phase drift and Jitter. In the Synch & Time budget CSP shall be added to SAT and LFAA.

3.2 Source L-0 Science Requirements:

PUID	Title	Requirement Text
SKA1-SCI_REQ-390	SKA1_Low timing accuracy	SKA1-LOW shall provide timing accuracy of at least 10 nsec over a 10 year time year interval to enable precision pulsar timing at 150 – 350 MHz (HPSO 5).
SKA1-SCI_REQ-388	SKA1_Low frequency accuracy	SKA1-LOW shall provide frequency accuracy of at least 10^{-11} over a 10 year interval. All applications using SKA1-LOW (e.g. HPSO 1 – 5) are otherwise negatively impacted, but particularly a measurement of the time evolution of cosmological red-shifts (RD1).

Table 1: Source L0 Science Requirements

3.3 Applicable Level-1 Requirements:

PUID	Title	Function	L0 Traceability	Applicable Budget
SKA1-SYS_REQ-3242	SKA1_Low coherence loss: 1second	Provide Reference frequency (Frequency dissemination)	SKA1-SCI_REQ-390	Coherence Loss budget
SKA1-SYS_REQ-3243	SKA1_ Low Coherence loss: 1 minute	Provide Reference frequency (Frequency dissemination)	SKA1-SCI_REQ-390	Coherence Loss budget
SKA1-SYS_REQ-3244	SKA1_Low Frequency reference phase drift	Provide Reference frequency (Frequency dissemination)	SKA1-SCI_REQ-390	Phase drift budget
SKA1-SYS_REQ-3616	UTC accuracy.	Provide Time (UTC time)	SKA1-SCI_REQ-388	Synch & Time Budget
SKA1-SYS_REQ-3617	Pulse per second precision	Provide Time (UTC time)	SKA1-SCI_REQ-388	Synch & Time Budget

SKA1-SYS_REQ-3280	SKA1-Low Central frequency reference	Provide Time (UTC time)	SKA1-SCI_REQ-388	Synch & Time Budget
SKA1-SYS_REQ-2956	SKA1_Low Time stamping	Provide Time (UTC time)	SKA1-SCI_REQ-388	Synch & Time Budget
SKA1-SYS_REQ-3294	SKA1_Low Common delay centre time	Provide Time (UTC time)	SKA1-SCI_REQ-388	Synch & Time Budget

Table 2 Applicable L1 Requirements

4 Coherence Loss Budget

4.1 Summary Description

4.1.1 Coherence Loss

[RD3] Phase fluctuations due to frequency instabilities (frequency accuracy) in the sampling clock and in the receiver signal path create a resultant phase fluctuation in the digitised signal.

In an interferometer, the visibility functions are composed by the cross-correlation product between 2 antennas, where both will suffer independently of the aforementioned phase fluctuations.

Is the relative phase fluctuation between the 2 antennas which defines the system phase fluctuation $\Delta\phi$.
Denoting $S_{12}(\omega_i)$ as the cross spectrum of the ideal correlation between 2 antennas (without phase fluctuations), then the cross spectrum of a system with phase fluctuations can be written as:

$$S'_{12}(\omega_i)e^{i\Delta\phi} \quad (1)$$

The resultant spectral power $S'_{12}(\omega_i)$ is smaller than in the ideal case. The coherence loss function can be defined as

$$C(T) = \left(1/T\right)\left|\int_0^T e^{i\Delta\phi}dt\right| \quad (2)$$

where $C(T)$ expresses the diminution from the unity due to incoherent averaging for integration period T. Loss of coherence will cause a proportional reduction in the (S/N) of the system. I.e.: 2% of coherence loss will decrease the S/N in 2%.

The coherence loss can be defined as

$$L_C = 1 - \sqrt{E[C^2(T)]} \quad (3)$$

The coherence function is related to the Allan variance by [RD4]:

$$\langle C^2(T)\rangle = \frac{2}{T}\int_0^T \left(1-\frac{\tau}{T}\right) exp\{-(\pi\upsilon_0)^2\left[\sigma_y^2(\tau)+\sigma_y^2(2\tau)+\sigma_y^2(4\tau)+\sigma_y^2(8\tau)+\ldots\right]\}d\tau \quad (4)$$

with τ the time interval between measurements and σ_y^2 the Allan variance of the fractional frequency instability.

4.1.2 Budget Definition

For the system budget the Coherence Loss is defined as the lack of coherence between the two signals of the SKA1_Low station pair being correlated. As a result of non-perfect correlation a reduction of the signal-to-noise ratio is observed. The Coherence Loss requirements apply to local interferometry. In SKA1_Low the Coherence Loss is dominated by oscillator (sampling clock) phase instabilities.

The coherence loss budget is defined as the maximum loss of coherence (in %) allowed during a correlator dump time. Loss of coherence decreases the Signal-to-Noise ratio for the correlated signal. This budget is a figure to be applied/verified on every single frequency distribution link and stations in the array.

The loss of coherence is proportional to the total phase error in the system (as an RMS value) due to instabilities and noise in the frequency distribution systems (STFR, LFAA), and the phase errors introduced by the ionosphere/atmosphere. Note that the clock instabilities are not part of the budget because the coherence loss is due to the relative phase difference and the clock sigma is common to the array elements.

4.2 Source L0 Requirements

PUID	Title	Requirement Text
SKA1-SCI_REQ-388	SKA1_Low frequency accuracy	SKA1-LOW shall provide frequency accuracy of at least 10^{-11} over a 10 year interval. All applications using SKA1-LOW (e.g. HPSO 1 – 5) are otherwise negatively impacted, but particularly a measurement of the time evolution of cosmological red-shifts

Table 3 L0 Requirements Driving Coherence Loss Budget

The requirement shall be understood as the capability of the telescope to measure the 1GHz redshift of 0.1 Hz with spectral resolution (or Uncertainty!) of 0.01Hz. This measurement depends on the survey speed of the telescope, the spectral resolution of the correlator and the stability of the reference frequency. This requirement cannot be satisfied by SKA1.

4.3 Applicable L1 Requirements

PUID	Title	Requirement Text
SKA1-SYS_REQ-3242	SKA1_Low coherence losses: 1s	The SKA1_Low Reference Frequency shall provide a 2% maximum coherence loss within a maximum integration period of 1 second and up to an operating frequency of 350 MHz
SKA1-SYS_REQ-3243	SKA1_Low coherence loss: 1minute	The SKA1_Low Reference Frequency shall provide a 2% maximum coherence loss for interval of 1 minute and up to an operating frequency of 350 MHz

Table 4 Applicable L1 Requirements to Coherence Loss Budget

4.4 Analysis Method

4.4.1 Relation to sensitivity on local interferometry

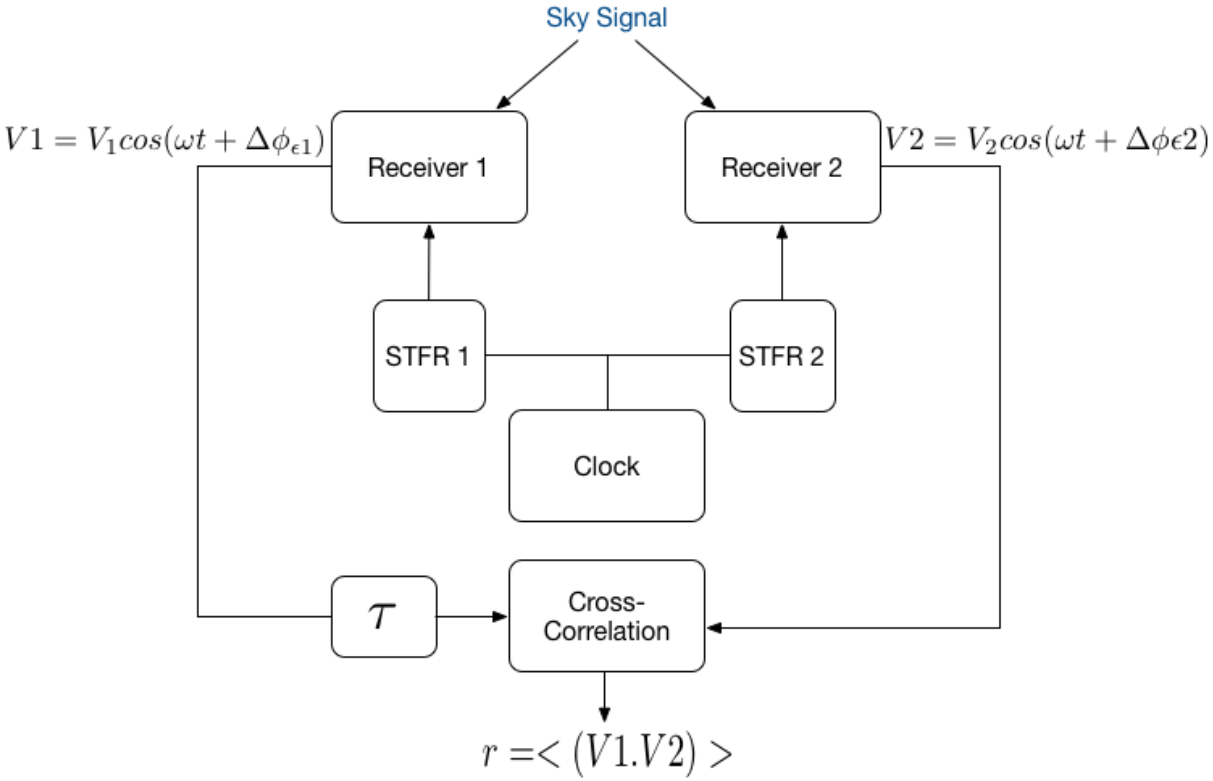


Figure 2 Local Interferometry for 2 Receivers with SADT.STFR Clock Distribution

Loss of coherence affects the sensitivity of the array which in turns affects the final science product. The figure 2 provides a simplified configuration for 2 receivers in interferometry. The reference frequency from the central clock is delivered by 2 independent STFR modules. The Sky signal is received, digitised and send to the correlator by both receivers.

The outgoing signals V1 and V2 are independent and characterised by a random phase instability $\Delta\phi_{\varepsilon i}$, it is assumed this instability per antenna has a RMS value $\Delta\phi_{\varepsilon i\ RMS}$.

The signal from antenna 1 is delayed by the instrumental delay τ before entering the correlator in order to compensate for the delay due to the geometrical signal path difference, then the cross-correlation is computed.

Assuming $V_1 = V_2 = V$, then the cross-correlation r, can be expressed as:

$$r(\tau) = \frac{V^2}{2} e^{j2\pi\upsilon\tau} C \quad (5)$$

with C the Coherence function. The Coherence function (and Coherence Loss) depends on $\Delta\phi_{\varepsilon i\ RMS}$.

As $\Delta\phi(t)$ is the resultant of the addition of $\Delta\phi_{\varepsilon 1}(t)$ and $\Delta\phi_{\varepsilon 2}(t)$ which are independent, then

$$\sigma^2_{\Delta\phi} = \sigma^2_{\Delta\phi_{\varepsilon 1}} + \sigma^2_{\Delta\phi_{\varepsilon 2}} \quad (6)$$

assuming that $\sigma^2_{\Delta\phi_{\varepsilon 1}} \approx \sigma^2_{\Delta\phi_{\varepsilon 1}} = \sigma^2_{\Delta\phi_{\varepsilon}}$, then the RMS value of the system phase noise can be written as:

$$\Delta\phi_{RMS} = \sqrt{2}\Delta\phi_{\varepsilon RMS} \quad (7)$$

with $\Delta\phi_{\varepsilon RMS}$ the RMS phase noise of a single receiver.

4.4.2 Phase fluctuations in Clock Distribution and Signal path

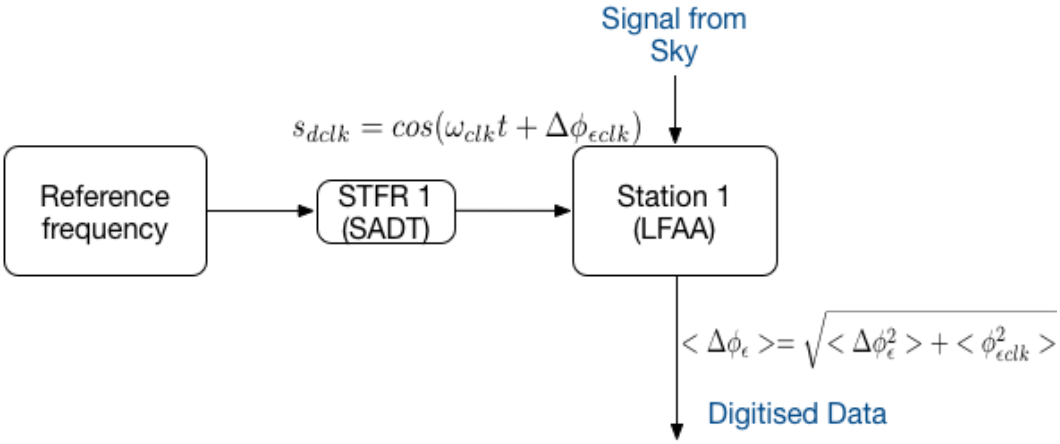


Figure 3 Phase Fluctuation Components in a Single Station

Figure 3 shows the actors that contribute to the phase fluctuations in a single station.

The resultant station phase fluctuation($\Delta\phi_\varepsilon$) is seen in the digitised signal and is composed of the phase fluctuation of the sampling clock ($\Delta\phi_{\varepsilon_{CLK}}$) and the phase fluctuation induced by the LFAA receiver electronics in the Sky signal path $(\Delta\phi_{\varepsilon_S})$. These contributions are added in quadrature (assuming they are independent variables). For this budgeting exercise the RMS value of the aforementioned phase fluctuations will be used. So the total phase fluctuation is written as:

$$\langle\Delta\phi_\varepsilon\rangle = \sqrt{\langle\Delta\phi^2_{\varepsilon_{CLK}}\rangle + \Delta\phi^2_{\varepsilon_S}} \quad (8)$$

The direct contributors to the sampling clock phase fluctuations are the signals delivered by the **STFR synthesizer (SAT system)** at the interface with LFAA and the LFAA **internal frequency distribution system** that delivers the sampling clock to the input of the digitiser.

The STFR delivers a 10 MHz reference frequency while LFAA distributes and synthesizes a 800 MHz sampling clock into the digitizer.

The behavior of the phase fluctuations depends on the kind of noise regime affecting the system, determining the noise regime in the system is out of the scope of this budget and is left to the consortia stakeholders to define it and derive the value of the phase fluctuations that will comply with the allocated coherence loss.

4.5 Calibration Steps and Effects

Coherence loss can be characterised (not calibrated) for every transfer link by measurement of the phase stability of the reference frequency. Alternatively, the coherence loss can be verified from astronomical verification, in a good case the atmospheric effects will be dominant over the reference frequency instabilities.

4.6 Summary of Margin and Residuals

Table 5 summarises the frequency stability system constraints set by the L1 requirements SKA1-SYS_REQ-2268 and SKA1-SYS_REQ-2692. These requirements specify a system 2% coherence loss for 1 second and 60 seconds.

As stated in the budget definition, the Coherence Loss constrains the system phase fluctuations and these fluctuations are $\sqrt{2}$ times the phase fluctuation of a single station. The value of the phase fluctuation of the contributors that complies with the coherence loss allocation is to be determined by the consortia stakeholders. The total phase fluctuation of the system will be calculated by the relationship defined in equation (8)

Maximum Sky Frequency (MHz)	350	
Requirement Number	Requirement	Coherence Loss (%)
SKA1-SYS_REQ-2268	Less than 2% coherence loss in 1 second	2
SKA1-SYS_REQ-2692	Less than 2% coherence loss in 1 min	2

Table 5 SKA1-Low System Coherence Loss and Frequency Stability

The coherence loss budget is initially allocated by SKAO based on a reasonable allocation approach. Consortia stakeholders will be able to propose different allocations based on inputs and evidence provided by them. The initial allocation is set in *table 6*, note the budget is focused in the worst case situation which corresponds to 350MHz.

STFR.FRQ @ 350 MHz		LFAA @ 350 MHz		Total Coherence Loss @ 350 MHz	
1s	**60s**	**1s**	**60s**	**1s**	**60s**
1.9%	1.9%	0.1%	0.1%	2.0%	2.0%

Table 6 SKA1-Low Coherence Loss Budget Allocation

4.6.1 Existing Risks

- Risk01: LFAA could fail to contribute less than 0.1% coherence loss

4.7 Source Errors

The following products are to be included as the main contributors in the budget:

CIN	Name	Details
	STFR.FRQ	Frequency Distribution
	LFAA	LFAA is treated as a single product that performs on coherence loss

Table 7 Main Contributors For SKA1-Low Coherence Budget

The following products are indirectly affecting the main contributor's products:

CIN	Name	Details
	SADT.LINFRA	Fiber optic links are affected by the environment. They will affect the performance of STFR.FRQ

Table 8 Secondary Contributors In SKA1-Low Coherence Budget.

4.8 Budget Maturity Table

Product (Allocation Target)	Assumption Value	Allocation Value	Margin	Actual Value (Analysis)	Actual Value (Test)	BCF
LFAA		0.1% (1sec)	0			
LFAA		0.1% (60sec)	0			
STFR.FRQ		1.9% (1sec)	0			
STFR.FRQ		1.9%(60sec)	0			

Table 9 SKA1-Low Coherence Loss Budget Maturity Table.

5 Phase Drift Budget

TBD1

6 Synchronisation and Time Budget

6.1 Summary Description

6.1.1 System Synchronisation and Time Budget

System Synchronisation and Time budget is related to the Provide Synchronisation, Time and Frequency function [RD2]. This function includes the timekeeping and the time measurement in the telescope, and the budget specifies the maximum uncertainty of these processes. The latter is critical for success of Pulsar Timing case and the study of gravitational waves. The SKA1 Low timescale has a single operation mode for all the observing modes, so the analysis is conducted only on the pulsar timing case as is the most stringent case. Pulsar timing requires measuring local time during observations with an uncertainty lower than 10ns over a 10 year timescale. Uncertainty levels require the timekeeping function not only to keep SKA time, but also to keep synchronisation to UTC time.

6.1.2 Budget Definition

This budget is defined towards the Pulsar Timing case as is the most stringent case. As the Synchronisation and Timing system (SAT) has a single operational mode, the other science modes (imaging, Pulsar search, VLBI) will inherit the performance from Pulsar Timing but they will have less risk regarding the satisfaction of the budget.

The Synchronisation and Timing budget specifies the maximum uncertainty of the *absolute time reference* used in the calculation of the Pulse Time of Arrival (TOA), section 6.4.1 provides a definition for this variable. The budget has traceability to Science and System requirements as listed in sections 6.2 and 6.3.

6.2 Source L-0 Requirements

PUID	Title	Requirement Text
SKA1-SCI_REQ-390	SKA1_Low timing accuracy	SKA1-LOW shall provide timing accuracy of at least 10 nsec over a 10 year time year interval to enable precision pulsar timing at 150 – 350 MHz (HPSO 5).

Table 10 L0 Requirements Driving Synchronisation and Time Budget

6.3 Applicable L1 Requirements

PUID	Title	Requirement Text
SKA1-SYS_REQ-3617	Pulse per second precision	The SKA1_Low shall have a 1 pps heartbeat signal precise to the sample rate (the pulse-to-pulse scatter shall be less than the inverse of the Nyquist frequency of the largest sampled bandwidth).
SKA1-SYS_REQ-3616	UTC uncertainty	The SKA1_Low timescales shall be traceable to UTC with an uncertainty of no more than 9 ns (1 σ).
SKA1-SYS_REQ-3280	SKA1-Low Central frequency reference	In order to avoid large offsets, the SKA1_Low Central Reference Frequency shall be steered to UTC to within at least 1 microsecond, with a frequency drift of less than 10 ns/day.

SKA1-SYS_REQ-2956	SKA1_Low Time stamping	For SKA1_Low sub arrays, each Pulsar Timing data sample shall be directly traceable to a time stamp corresponding to its common delay centre time.
SKA1-SYS_REQ-3294	SKA1_Low Common delay centre time	Each SKA1_Low subarray shall have a common delay centre at or near its centre with a time accurate to the SKA timescale and a precision of better than 2ns (1σ) over periods of one observation and at least 10 years.

Table 11 Applicable L1 Requirements

6.4 Analysis Method

6.4.1 Pulsar Timing

Pulsar Timing aims to measure the Time of Arrival (TOA) of the pulse. Figure 4 from [RD5] presents a typical scheme for Pulsar Timing processing.

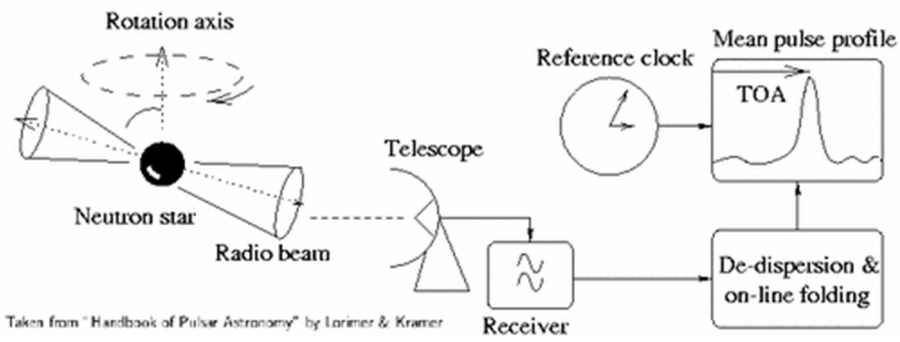


Figure 4 Pulsar Timing Process

[RD6] describes the Pulsar Timing process which can be summarised as a series of pulses is de-dispersed and averaged to obtain a high signal-to-noise average pulse profile. Typically, the average profile is correlated with a template or model profile so that its phase offset can be determined. Multiplying the phase offset by the pulse period yields the offset time t_{TOA}.

The pulse profile has a high-precision (low uncertainty) absolute time reference t_0, i.e.: (sic) the left edge of the profile based on the recorded time of the first sample of the observation.

TOA as absolute time is the addition of t_0 and t_{TOA}:

$$TOA = t_0 + t_{TOA} \quad (1)$$

Calling σ_0 the precision (uncertainty) of t_0, and σ_{TOA} the precision of the offset time t_{TOA}, then the uncertainty of the Time of arrival as absolute time is:

$$\sigma^2 = \sigma^2_0 + \sigma^2_{TOA} \quad (2)$$

Ideally for the science case of Pulsar Timing for detection of Gravitational Waves, σ should be less than 10ns. The contributors to σ^2 as outlined in (2) are now analysed. It shall be noted that the residual error of the Delay Model will be a contributor that could affect either the precision of σ^2_0 and $\sigma^2_{TOA.}$

6.4.1.1 Precision of the offset time: σ_{TOA}

Specification of this variable is not in the scope of the budget, but definition is given for reference and context of the analysis.

σ_{TOA} is usually expressed in terms of the width of the pulse feature W_f in units of the period P, the pulse period P, and the signal-to-noise ratio SNR

$$\sigma_{TOA} \propto W_f P \,/\, \text{SNR} \quad (3)$$

From (3) can be appreciated that σ_{TOA} depends on the observed pulsar characteristics therefore for the general case it may not be possible to constrain by design the maximum value of σ_{TOA}. Consistently, there is no L1 requirement constraining this parameter.

Residual delay errors will affect the precision by the reduction of the SNR, this effect is not analysed in this version of the document, but it could be included in future versions.

6.4.1.2 Precision of the absolute time reference: σ_0

Requirement SKA1-SYS_REQ-2764 defines that the absolute time (timestamp) associated to a data sample will correspond to the absolute time as measured/estimated at the Common Delay Centre (Array Phase Centre). Requirement SKA1-SYS_REQ-3293 indicates this time shall have a precision better than 2ns with respect to the SKA Timescale.

Requirement SKA1-SYS_REQ-2274 indicates the SKA Timescale shall have an uncertainty better than 5ns compared to UTC. With this, the precision of t_0 will be made by the squared addition of the Timescale uncertainty, σ_{SKA}, and the precision of the Common Delay Centre to the Timescale σ_{CDC}.

$$\sigma^2_0 = \sigma^2_{SKA} + \sigma^2_{CDC} \quad (4)$$

σ^2_{SKA} is considered to have a single contributor which is the SKA Timescale itself, on the other hand σ^2_{CDC} will have contributors in SADT and LFAA.

As mentioned in the previous section, for Pulsar Timing, the absolute time reference t_0 can be defined as the recorded time of the first sample of the observation. Without prescribing a design solution, a base case can be defined by estimating t_0 from the timestamps of the samples coming from the receivers and the corrections applied by the delay model.

6.4.2 Measurement of t_0 From the Timestamps of the Samples Coming From the Receivers and the Delay Model

Note: **This analysis is not intended to prescribe any design solution to the consortia.**

Figure 5 describes the architecture required to measure t_0 from the timestamps of the samples from the receivers.

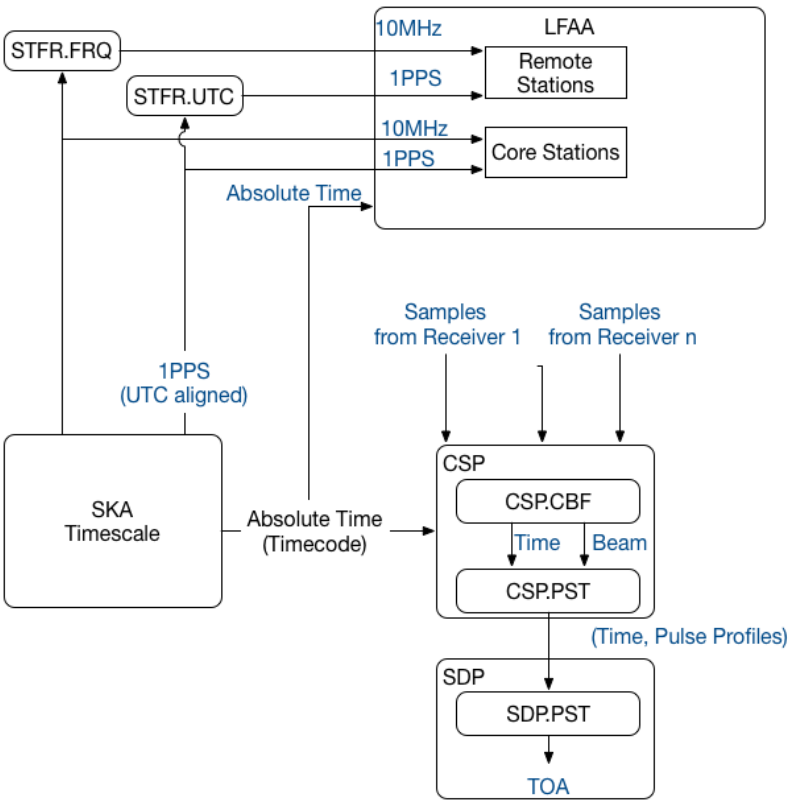


Figure 5 Telescope Architecture for Pulsar Timing with t0 Measurement From the Receivers Timestamps

For this analysis the following assumptions are done:

- The SKA Timescale produces the time t_{SKA} with offset to UTC μ_{SKA} and uncertainty σ_{SKA}
- The SKA Timescale and LFAA internal timing distribution systems of Core Stations are in the same building (CPF).
- The environment inside the building is controlled and stable so system performance is stable and predictable (No sensible effect of environment changes).
- The 1PPS is distributed to LFAA Core Stations via a direct connection from the Timescale.
- The 1PPS physical connection introduces a delay δ_c and noise σ_c
- σ_c can be assumed to be negligible due to the location and environmental condition assumptions.
- LFAA receives the time of the 1PPS as a code representing an integer second, note this is the SKA time, t_{SKA}, or UTC(SKA).
- LFAA internally distributes the 1PPS using internal products.
- The LFAA modules introduce a 1PPS delay δ_{LFAA_PPS} and noise σ_{LFAA_PPS}.
- The LFAA Field Nodes in Core Stations distribute the sky signal over an RFoF link that introduces delay $\delta_{LFAA_CS_RFoF}$ and noise $\sigma_{LFAA_CS_RFoF}$.
- The LFAA Field Nodes in Remote Stations distribute the sky signal over an RFoF link that introduces delay $\delta_{LFAA_RS_RFoF}$ and noise $\sigma_{LFAA_RS_RFoF}$.
- The Stations performs beamforming with a station beam residual error σ_{LFAA_SB}.
- δ_c, δ_{LFAA_PPS}, δ_{LFAA_RFoF}, σ_{LFAA_PPS}, σ_{LFAA_RFoF} and σ_{LFAA_SB} can be characterised.
- The STFR.UTC distributes 1PPS to the LFAA Remote Stations using the White Rabbit protocol and introduce a delay δ_{WR} and noise σ_{WR}.
- LFAA Core and Remote stations will timestamp the digitised data by reading the time of the received 1PPS signal and counting the number of samples after the 1PPS.

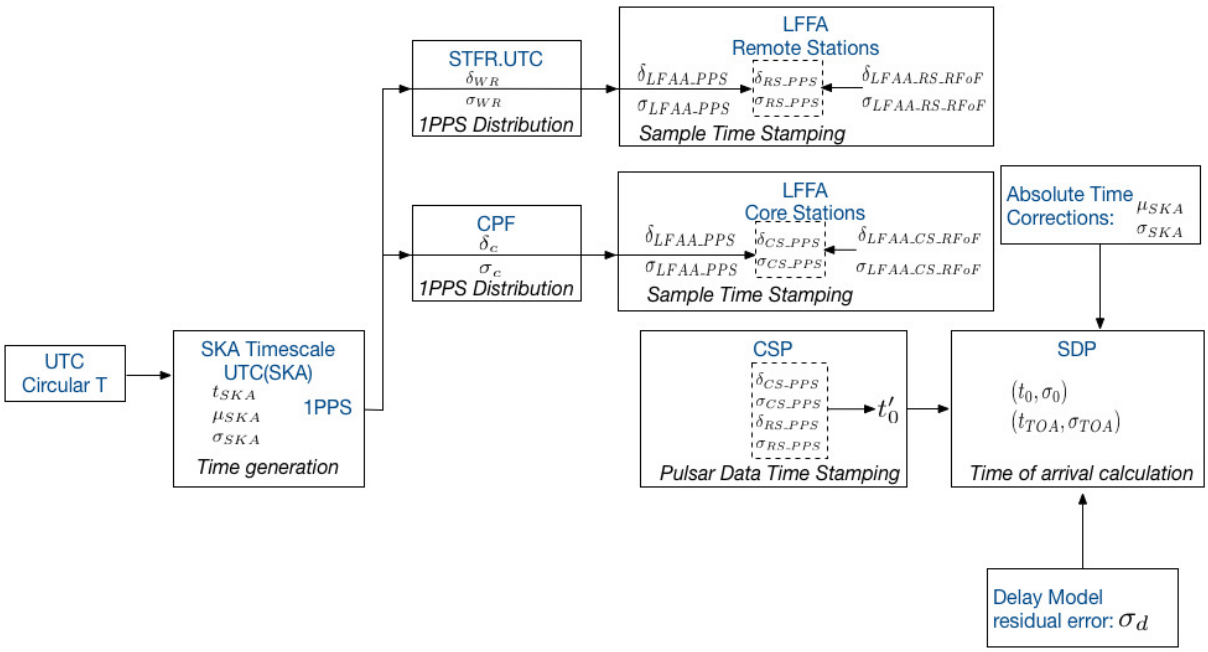


Figure 6 Delay and Uncertainty Variables For the Analysis (To be Completed)

Figure 6 displays the delay and uncertainty variables used for the following analysis.

For the Core Stations, the 1PPS signal gets to the LFAA interface delayed by δ_c and then is internally distributed into the digitisation stage with an additional delay δ_{LFAA_PPS}. Then, the total 1PPS delay for the Central Stations, δ_{PPS_CS}, is:

$$\delta_{CS_PPS} = \delta_c + \delta_{LFAA_PPS} \quad (5)$$

The uncertainty of the time at the distributed 1PPS with respect to the Timescale is:

$$\sigma_{CS_PPS} = \sigma_{LFAA_PPS} \quad (6)$$

In the Remote Stations, the 1PPS signal delivered by the White Rabbit system arrives with delay δ_{WR} and then is internally distributed into the digitisation stage with an additional delay δ_{LFAA_PPS}. Then, the total 1PPS delay for the Remote Stations, δ_{RS_PPS}, is:

$$\delta_{RS_PPS} = \delta_{WR} + \delta_{LFAA_PPS} \quad (7)$$

The uncertainty with respect to the Timescale of the time at the Remote Station 1PPS is:

$$\sigma^2_{RS_PPS} = \sigma^2_{WR} + \sigma^2_{LFAA_PPS} \quad (8)$$

The LFAA Core Stations will timestamp the k^{th} digitised samples after the 1PPS signal with sample rate N at time t_{CS},

$$t_{CS} = t_{SKA} + \delta_{CS_PPS} + k/N \quad (9)$$

(9) shall be corrected to account for the RFoF link delay, so the time of the signal is

$$t_{CS} + \delta_{LFAA_CS_RFoF} = t_{SKA} + \delta_{CS_PPS} + k/N \quad (10)$$

Accounting for the residual error of the Station Beamforming, and assuming the components behave independently, the uncertainty of the timestamp is

$$\sigma^2_{CS} = \sigma^2_{LFAA_PPS} + \sigma^2_{LFAA_CS_RFoF} + \sigma^2_{LFAA_SB} \quad (11)$$

The jitter of the sampling clock shall be in the range of sub picoseconds (see Jitter Budget), thus is negligible for this analysis.

For the Remote Stations similar relationships for the timestamps are defined:

$$t_{RS} + \delta_{LFAA_RS_RFoF} = t_{SKA} + \delta_{RS_PPS} + k/N \quad (12)$$

$$\sigma^2_{RS} = \sigma^2_{WR} + \sigma^2_{LFAA_PPS} + \sigma^2_{LFAA_RS_RFoF} + \sigma^2_{LFAA_SB} \quad (13)$$

It is possible to make an A Priori assumption that the RFoF links for the Remote Stations will be much more shorter than the links for the Core Station, therefore the contributions to uncertainty of the RFoF links in the Remote Stations could be much lower than the expected contribution to uncertainty of the Core Stations. At the moment there is not information available to confirm this assumption and to resolve which ones will have the lowest uncertainty. For simplicity for numeric calculations $\sigma_{LFAA_CS_RFoF}$ and $\sigma_{LFAA_RS_RFoF}$ will be treated as having similar values (this will be updated in future versions when more detailed information is available).

Next, the digitised data are sent by LFAA to CSP and the Delay Model is applied to align the samples. The set of the timestamps from the LFAA stations can be corrected with the set of delays δ_d from the Delay Model.

Assuming the Delay Model is accurate with a residual error σ_d , then the set of the corrected timestamps will converge to represent the time of the beam and/or the time of the Common Delay Centre. The level of residual error is estimated to be near to $\lambda/10$ of the observed wavelength, which corresponds to 1/10 of the period of the equivalent observed frequency.

The uncertainty of the Common Delay Centre time, σ_{CDC}, with respect to the SKA Timescale depends on the dispersion value of the corrected timestamps and the residual errors of the Delay Model σ_d . If there are *m* Core Stations and *n* Remote Stations, then σ_{CDC} can be expressed as:

$$\sigma^2_{CDC} = (m\sigma^2_{CS} + n\sigma^2_{RS})/(m+n) + \sigma^2_d \quad (14)$$

Replacing in (14) σ_{CS} and σ_{RPF} from (11) and (13), it is obtained

$$\sigma^2_{CDC} = \sigma^2_{LFAA_PPS} + (m\sigma^2_{LFAA_CS_RFoF} + n\sigma^2_{LFAA_RS_RFoF})/(m+n) + \sigma^2_{LFAA_SB} + n\sigma^2_{WR}/(m+n) + \sigma^2_d \quad (15)$$

Requirement SKA1-SYS_REQ-3294 constrain σ_{CDC} to be lower than 2ns.

The reference time t_0 can be defined in different ways from the set of corrected timestamps, i.e.: the mean of the timestamps or the median of the timestamps.

A corrected timestamp as absolute time will be made by

$$t_{stamp} = t_{SKA} + \delta_{PPS} - \delta_{LFAA_RFoF} + \delta_d + k/N \quad (16)$$

In (16) δ_{PPS} and δ_d will exhibit an statistical behaviour, therefore if δ^* is the implemented statistical estimator of the delays, then t_0 can be expressed as:

$$t'_0 = t_{SKA} + \delta^*_{PPS} - \delta^*_{LFAA_RFoF} + \delta^*_d + k/N \quad (17)$$

Lately t_{SKA} , the UTC realisation of SKA, can be traced to UTC from the BIMP Circular T report which indicates the offset (μ_{SKA}) and the uncertainty (σ_{SKA}) of the SKA Timescale. The corrected absolute time term t_0 can then be expressed as:

$$t_0 = t'_0 + \mu_{SKA}$$

$$t_0 = t_{SKA} + \mu_{SKA} + \delta^*_{PPS} - \delta^*_{LFAA_RFoF} + \delta^*_d + k/N \quad (18)$$

The uncertainty of t_0 is given by (4), replacing σ^2_{CDC} with (15) results :

$$\sigma^2_0 = \sigma^2_{SKA} + \sigma^2_{LFAA_PPS} + (m\sigma^2_{LFAA_CS_RFoF} + n\sigma^2_{LFAA_RS_RFoF})/(m+n) + \sigma^2_{LFAA_SB} + n\sigma^2_{WR}/(m+n) + \sigma^2_d \quad (19)$$

with σ_{SKA} the SKA Timescale uncertainty to UTC as reported by BIPM. The 6 terms to the right of the equation are to be constrained by the 2ns condition (σ^2_{CDC}<2ns), therefore the budget in this case will have allocations to LFAA, SADT and the Delay Model.

Is interesting to observe that the factor n/(m+n) which is multiplying σ^2_{WR} reduces the impact of the SADT Timing dissemination system in the total uncertainty.

6.4.3 t_0 Uncertainty Contributors

Table 12 shows the contributors for the uncertainty of the t_0 estimations as described by (14):

Contributor	
SKA Timescale	σ_{SKA}
LFAA RFoF link (Core Stations)	$\sigma_{LFAA_CS_RFoF}$
LFAA RFoF link (Remote Stations)	$\sigma_{LFAA_RS_RFoF}$
LFAA Station Beam	σ_{LFAA_SB}
LFAA 1PPS internal distribution	σ_{LFAA_PPS}
STFR.UTC 1PPS distribution	σ_{WR}
Delay Model	σ_d

Table 12 t_0 uncertainty contributors

From a system budget perspective, the SKA Timescale is allocated with 9ns from requirement SKA1-SYS_REQ-3616. The functions of Time distribution/dissemination and Delay Model have to share the allocation of 2ns from requirement SYS_REQ-3294.

6.5 Calibration Steps and Effects

Calibration is required at different stages:

- The SKA Timescale which follows a calibration cycle through the monthly loop with BIPM. Every month BIPM will report the offset (μ_{SKA}) and uncertainty (σ_{SKA}) of the SKA Timescale, as a UTC realisation, with respect to UTC.
- 1PPS distribution systems in SADT require a characterisation of the propagation delay and the precision degradation (or uncertainty) added to the 1PPS signal. These systems shall procure a stable and predictable performance. It shall be noted that there are 108 stations of a total of 512 using the SADT system, this implies that only 21% of the SADT uncertainty contribution will affect the total uncertainty.
- Residual errors of the Delay Model will affect the reference time measurement and the estimation of the Time of Arrival. This factor is directly related with the telescope calibration. It is estimated that the RMS of residual errors will be approximately $\lambda/10$ of the observed wavelength, which corresponds to 1/10 of the period of the equivalent observed frequency. The lowest frequency observed for Pulsar Timing with SKA1-Low is around 200 MHz, which gives a worst case for the residual error of σ_d = 0.5ns, to allow some flexibility this budget will consider σ_d = 0.75ns.

6.6 Summary of Margin and Residuals

Table 13 summarises the allocation of maximum allowed uncertainty made by the system contributors. In this case the timescale is directly allocated with 9ns; while the time dissemination (for both SADT and LFAA) and Delay Model are allocated with 2 ns.

Table 14 shows the allocations for the contributions for the 1PPS dissemination and the delay model and the total uncertainty calculated with equation (15) and n = 108 and m + n = 512, for simplicity $\sigma_{LFAA_CS_RFoF}$ and $\sigma_{LFAA_RS_RFoF}$ are assumed to have the same value. This budget does not set the internal budget for LFAA components, so allocation is done at the level of LFAA, is up to the consortium to set an internal budget that respects this allocation .

	Time Scale	1PPS dissemination (SADT+LFAA) + Delay Model	Total Uncertainty	Margin
Uncertainty [ns]	9	2	9.22	0.78

Table 13 SKA1-Low Synchronisation and Time Budget Allocation

	SADT: STFR.UTC 1PPS distribution	LFAA	Delay Model	Total Uncertainty	Margin
Uncertainty [ns]	1.5	1.5	0.75	1.81	0.19

Table 14 Timing Dissemination and Delay Model Uncertainty Allocations

Note the LFAA allocation includes σ_{LFAA_PPS}, $\sigma_{LFAA_CS_RFoF}$, $\sigma_{LFAA_RS_RFoF}$ and σ_{LFAA_SB} , the allocation assumes that they behave independently such that:

$$\sigma^2_{LFAA_PPS} + (m\sigma^2_{LFAA_CS_RFoF} + n\sigma^2_{LFAA_RS_RFoF})/(m+n) + \sigma^2_{LFAA_SB} <= 1.5 \text{ ns}.$$

There is a risk that the independence assumption is not true.

6.6.1 Existing Risks

- Issue:
    - Risk02: Combined effects of SADT and LFAA 1PPS distribution could exceed the 2 ns allocation leaving no margin for the Delay Model error.
    - Risk03: LFAA 1PPS distribution design maturity is low. It is not possible at the moment to foresee its performance specification.
    - Risk04: Delay model expected performance in terms of RMS residual error could be much worse than $\lambda/10$. For frequencies in the order of 50MHz is not possible to comply with the budget.
    - Risk05: It Is assumed that LFAA components behave independently
    - Risk06: The analysis assumes there is no systematic errors. This could be an issue for long term performance.
    - Risk07: Allocation for SADT is very close to the expected worst case uncertainty, the small margin could imply that overall 2ns system budget is not satisfied.

6.7 Source Errors

Product (PBS/CIN)	Consortia/Owner	Analysis Report	Rev/Date
SAT.Timescale.SKA1-Low / 141-0400000	SADT		
SAT.STFR_UTC_SKA1_Low / 141-0300000	SADT		
LFAA. SPS / 165-020000	LFAA		
LFAA.Field Node / 160-000000	LFAA		
Delay Model			

Table 15 SKA1-Low Synchronisation and Time Budget Contributors

6.8 Budget Maturity Table

Product (Allocation Target Product)	Assumption Value	Allocation Value	Margin	Actual Value (Analysis)	Actual Value (Test)	BCF
SAT.Timescale.SKA1-Low		9ns	0			
SAT.STFR_UTC_SKA1_Low		1.5				
LFAA		1.5				
Delay Model		0.75				

Table 16 SKA1-Low Synchronisation and Time Budget Maturity Table

7 Jitter (Rev.2)

7.1 Summary Description

Jitter specifies the Sampling Clock quality for phase noise performance. For Jitter there are no L1 requirements related to timing but spectral and digitization requirements for LFAA can specify jitter. This second revision for the Jitter budget in Low aggregates a discussion for the SNR in the telescope and Jitter specification that justifies to update the working assumptions and derive a more practical definition for the budget.

7.2 Source L0 Requirements

Not applicable.

7.3 Applicable L1 Requirements

There are not L0 or L1 requirements directly defining jitter, however specifications of bandwidth and number of bits, allows for a definition of jitter from L1 requirements.

PUID	Title	Requirement Text
SKA1-SYS_REQ-2147	Instantaneous Bandwidth	300MHz of bandwidth is available when the SKA1_Low is producing just a single beam per station. Multiple beams have an aggregate B/W of 300MHz. Even if the beam(s) are sparsely populated with receive channels.
SKA1-SYS_REQ-2674	Digitisation	Digitisation of SKA1_Low antenna signals shall be to at least 8 bits.

Table 17: Applicable L1 Requirements

7.4 Analysis Method

7.4.1 Definitions

[RD8] provides the following definitions applicable in the analysis for jitter.

7.4.1.1 ***Signal to Noise Ratio (SNR):***

The ratio of the rms signal amplitude to the mean value of the root-sum-squares (RSS) of all other spectral components, excluding the first 5 harmonics and DC.

7.4.1.2 ***N-bit Converter SNR****:*

For an ideal N-bit converter the SNR due to quantization noise is

$$SNR = 6.02N + 1.76\ dB \quad (7.1)$$

7.4.1.3 ***Signal to Noise and Distortions (SINAD):***

The ratio of the rms signal amplitude to the mean value of the root-sum-squares (RSS) of all other spectral components, including harmonics, but excluding DC.

Note: Maybe is not the most accurate application for the SINAD definition but for simplification in this discussion it will be considered for SINAD to also include the effects of Quantization Noise, Effective Input noise and Differential Nonlinearity (DNL).

*7.4.1.4 **Effective Number of Bits (ENOB):***

$$ENOB = \frac{SINAD[dB] - 1.76}{6.02} \quad (7.2)$$

*7.4.1.5 **SNR Due to Jitter**:*

For an ideal ADC with infinite resolution the resulting SNR due to jitter is

$$SNR[dB] = 20log_{10}\frac{1}{2\pi f_i t_j} \quad (7.3)$$

with f_i the input analog frequency and t_j is the time jitter.

Note: These application notes are written from the ADC manufacturer's point of view, with the assumption that the device will be operated with full-scale input signal, the worst-case condition for their specifications. In the SKA case, these formulas should be evaluated based on the effective dynamic range of the input signal, not the converter bit depth.

*7.4.1.6 **SNR Due to Jitter for a Flat Spectrum***

Equation (3) is derived assuming a sinusoid input, for a flat spectrum with center frequency $f_0$and bandwidth BW, the SNR due to jitter t_j can be expressed as

$$SNR[dB] = 20log_{10}\frac{1}{2\pi\,(f_0 + \frac{BW}{\sqrt{12}})} \quad (7.4)$$

if $f_0 \gg BW$ then the pure sinusoid case is obtained.

Note: (4) is not presented in "Analog-Digital Conversion Handbook (2004)". This equation may offer a different criteria to assess the SNR and the jitter effects, however it shall be understood that the spectrum input in Low will not be flat, so as (3) is also an approximation.

7.4.2 Discussion on the SNR for SKA1-Low

SKA1-Low needs high SNR and accuracy after long integrations. However, on individual samples it is acceptable to have 0.1 to 0.5% of added noise (equivalent to SNR ~25 to 30 dB). The digitization process should not clip with an input of Gaussian random noise and have extra room for RFI, therefore 8 bits digitisation is needed. A system with ENOB ~8 has a SINAD of about 50 dB (note input amplitude is not necessarily Full-Scale).

The expected dynamic range of the signals to be captured by SKA1-Low is, based on real measurements of the RF spectrum on site, ~8 bits. This value is an over-estimation because the dominant power in the band in from only 5 strong signals at frequencies below 100MHz, and the

amplitude at 350MHz is much lower. It is not considered that the input signal will contain Full-Scale signals at any one frequency - if it does it will be RFI flagged and the data will not be used. On the other hand, a system with 10+ bits is not affordable within our budget and today's technology, and is not required on the current scientific requirements.

As bottom line, from the previous paragraph, assuming the signal dynamic range to be 8 bits but knowing the spectral band composition has no full-scale signals, and that the amplitude at 350 MHz is smaller than 100 MHz, and that the dominant power comes from signals below 100 MHz; then it can be taken as directive to take a design goal for the digitisation to perform with ENOB of 8 bits, however is understood that this value has not to be strictly satisfied and small deviations around it can be accepted.

The existing jitter specification allows the system to have SINAD of ~72 dB with negligible jitter noise (ENOB ~12 bits), which is by far an over specification for a system requiring to perform to 8 bits.

7.4.3 Discussion on Initial Jitter Specification

As reference the jitter in LOFAR is 350 fs in a higher RFI environment (specification is 500 fs).

Worst case assumptions include making Input signal amplitude equal to the ADC Full-Scale value. Required SNR is much smaller than the Dynamic Range provided by 12 or 14 bits ADC, so keeping this assumption is a mistake and created a gold-plated specification for jitter.

500 fs jitter makes jitter noise dominate within a 12-bit system but is good for ~10 ENOB, which is easier and cheaper and is similar to what was done for LOFAR.

All these inputs are evidence that 75 fs jitter allocation to LFAA (and 110 fs for the system) is a wrong specification.

7.4.4 Updated Assumptions

From the arguments exposed in this discussion the assumptions for the jitter budget are updated as follow:

- Signal dynamic range is 8 bits. This includes the signals of interest, 95% of maximum RFI and prevents Gaussian noise clipping.
- SINAD includes all distortion and noise as described in the SINAD definitions (Except the jitter noise). The analysis is agnostic to the specifications of the ADC.
- For simplicity SINAD is assumed to be 50 dB for the ENOB calculation (SINAD = 49.92 dB gives ENOB = 8.0).
- ENOB of 8 bits is the performance goal for the design. However this value does not need to be strictly satisfied in the system realization and small deviations (~0.1bits) can be accepted in order to trade-off for a feasible and affordable system.
- Maximum input frequency is 350 MHz
- Input signal amplitude in the ADC is not Full-Scale.
- JItter is defined as RMS value.
- The sampling clock generation and distribution is done in the following way
    - SAT generates a 10 MHz reference signal at the SKA Timescale (Clock)
    - SAT distributes and delivers the 10 MHz signal to LFAA

        - For core stations interface to LFAA is with every SPS rack at the CPF
        - For remote stations interface is with SPS rack at every RPF
        - The Jitter at this interfaces is equivalent to the SAT (SADT) contribution to the System Jitter and is SADT jitter specification for the SADT-LFAA ICD
    - From the LFAA SPS racks the 10 MHz signal is distributed to the TPMs
    - Within the TPM the signal follows the Clock path, is upconverted to 800MHz and reaches the ADC clock input.
        - The jitter of the clock signal at the ADC clock input is the System Jitter
        - The jitter added from the SPS rack input to the ADC clock input is the LFAA contribution to the System jitter
- SADT and LFAA jitter contributions add in quadrature
- The jitter contributions are allocated by the jitter budget

7.4.5 Jitter

Taking equation (3), with $f_i = 350$ MHz and $t_j = 400$ fs, then SNR_{jitter} for an ideal ADC will be approximately 61 dB.

SINAD can be combined with SNR_{jitter} as:

$$SNR = -10log_{10}\left(10^{\frac{-SINAD}{10}} + 10^{\frac{-SNR}{10}}\right) \quad (7.5)$$

From (7.5), combining the assumed SINAD value of 50 dB with the SNR_{jitter} of 61 dB gives a total SNR of ~49.7 dB.

Calculating SNR_{jitter} with (7.4) and bandwidth of 300 MHz, makes SNR_{jitter} go to 59.2 dB and total SNR to ~49.6 dB.

With these values, taking (2), ENOB will drop in around 0.05 bits.

If the signal amplitude at 350 MHz is even smaller it will make the jitter noise negligible. Increasing Jitter to 500 fs **will** drop ENOB in about 0.08 bits.

From this analysis it is proposed to update the budget for a total jitter of 400 fs and include margin for up to 500 fs.

7.5 Calibration steps and effects

For science data will be important to know the ENOB rather than the jitter level. The jitter level has to be constrained to have the optimum performance of the ADC and use its entire resolution.

The Sampling Clock Jitter can be determined by measuring at the ADC clock input with an osciloscope or with a phase noise measurement instrument. If this point is not available, then measuring the Reference frequency shall give an accurate estimator of the clock jitter. On operation of the ADC, the jitter will affect the ENOB, so counting the number of noisy bits allows to estimate the jitter level; ideally the system has to be adjusted to have only the LSB as a noise bit.

One way to the check for the ADC operation and the jitter level consists into feeding the ADC with a stable signal (both amplitude and frequency), make the digitization and then obtain the FFT; from the

FFT an spectral analysis can be done and determine the level of SNR and spectral artifacts added by the ADC and sampling clock combination.

7.6 Summary of Margin and Residuals

The margin philosophy is given by the maximum allowed jitter for the worst case for a given ENOB as defined by equation (7.3). The ENOB will be considered to be 8 bit, the will consider that this value could deviate in up to 0.1 bits.

A 500 fs jitter will reduce ENOB in 0.08 bits. This budget will define a system jitter of 400 fs and will provide margin for up to 500 fs total.

Allocations to SADT and LFAA will be equal.

	SADT.STFR.FRQ	Tile Processing Module	Total Jitter	Margin
Jitter [fs rms]	283	283	400	300

Note: Total Jitter and Margin Add in Quadrature.

7.6.1 Existing Risks

- Risk08: The current analysis assumes a sub-optimal configuration, where both SADT and LFAA may be forced to implement cascaded jitter reduction mechanisms (Cleaning Oscillator), to achieve the specified budget allocation. In the current phase of design is not possible to resolve an optimal system solution, so this will forwarded to the Bridging Phase, where this problem will be seen to optimise the configuration and update this budget.

7.7 Source Errors

Product (PBS/CIN)	Consortia/Owner	Analysis Report	Rev/Date
SAT.STFR.FRQ.THU / 141-011000	SADT	[RD4] Page 31	Rev04/08-2015
Tile Processing Module / 161-000000	LFAA		

7.8 Budget Maturity Table

Product (Allocation Target Product)	Assumption Value	Allocation Value	Margin	Actual Value (Analysis)	Actual Value (Test)	BCF
SAT.STFR.FRQ		283 fs	0			
Tile Processing Module + LFAA Frequency distribution		283 fs	0			

8 Outstanding Issues, Risks, TBDs and TBCs

Id	Affected Budget	Name	Description	Owner	Resolution date
TBD1	Phase Drift Budget	Phase Drift budget	Complete Phase drift budget	SKAO	2018 Q3
Risk01	Coherence Loss	LFAA Coherence Loss	LFAA could not achieve the budget allocation		
Risk02	Synchronisation and Time Budget	2 ns performance	Combined effects of SADT and LFAA 1PPS distribution with could exceed the 2 ns allocation leaving no margin for the Delay Model error.		
Risk03	Synchronisation and Time Budget	LFAA 1PPS distribution	LFAA 1PPS distribution design maturity is low. It is not possible at the moment to foresee its performance specification.		
Risk04	Synchronisation and Time Budget	Delay Model performance	Delay model expected performance in terms of RMS residual error could be much worse than $\lambda/10$. For frequencies in the order of 50MHz is not possible to comply with the budget.		
Risk05	Synchronisation and Time Budget	Assumption of LFAA components independent behaviour	It Is assumed that LFAA components behave independently, if this is wrong there will be systematic errors not accounted in the budget		
Risk06	Synchronisation and Time Budget	Systematic Errors	The analysis assumes there is no systematic errors. This could be an issue for long term performance.		
Risk07	Synchronisation and Time Budget	SADT allocation	Allocation for SADT is very close to the expected worst case uncertainty, the small margin could imply that overall 2ns system budget is not satisfied.		
Risk08	Jitter Budget	Sub-optimal assumption analisis	The current analysis assumes a sub-optimal configuration, where both SADT and LFAA may be forced to implement cascaded jitter reduction mechanisms (Cleaning Oscillator), to achieve the specified budget allocation. In the current phase of design is not possible to resolve an optimal system solution, so this		

			will forwarded to the Bridging Phase, where this problem will be seen to optimise the configuration and update this budget.		